

Hong Kong Mathematics Olympiad (2018/19)
Final (Individual – Event 1)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. If A is the coefficient of x^4 in the expansion of $(x^2 + 2)^5$, determine the value of A .
若 A 是 $(x^2 + 2)^5$ 展開式中 x^4 的係數，求 A 的值。

$A =$

2. If x and y are positive integers that satisfy

$$\log_{10} x + \log_{10} y = \log_{10} (2x - Ay) + 1,$$

and B is the number of possible pairs of (x, y) , determine the value of B .

若 x 和 y 為正整數，並且滿足以下等式

$$\log_{10} x + \log_{10} y = \log_{10} (2x - Ay) + 1,$$

而 B 是 (x, y) 所有可能組合的數量，求 B 的值。

$B =$

3. If $Y = 2^{3(B-1)}$ and C is the sum of the digits of Y , determine the value of C .

若 $Y = 2^{3(B-1)}$ 並且 C 是 Y 中每個數字之和，求 C 的值。

$C =$

4. In $\triangle XYZ$, $XY \perp YZ$, $\angle XZY = \theta$, and the lengths of YZ and XZ are $C - 3$ and $C + 5$, respectively. If $D = (\sin \theta + \tan \theta)^2$, determine the value of D .

在 $\triangle XYZ$ 中，已知 $XY \perp YZ$ ， $\angle XZY = \theta$ ，線段 YZ 和 XZ 的長度分別為 $C - 3$ 和 $C + 5$ 。若 $D = (\sin \theta + \tan \theta)^2$ ，求 D 的值。

$D =$

Hong Kong Mathematics Olympiad (2018/19)
 Finals (Individual – Event 2)

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Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. If $\sqrt{A} = \sqrt{11+\sqrt{21}} - \sqrt{11-\sqrt{21}}$, determine the value of A .
若 $\sqrt{A} = \sqrt{11+\sqrt{21}} - \sqrt{11-\sqrt{21}}$ ，求 A 的值。

$A =$

2. If the straight line $y = mx + B$ passes through the two points $(4, 5)$ and $(-A, A)$, determine the value of B .
若直線 $y = mx + B$ 經過兩點 $(4, 5)$ 和 $(-A, A)$ ，求 B 的值。

$B =$

3. If $\cos x + \sin x = \frac{2B}{5}$ and $C = (\tan x + \cot x)^{-1}$, determine the value of C .
若 $\cos x + \sin x = \frac{2B}{5}$ 及 $C = (\tan x + \cot x)^{-1}$ ，求 C 的值。

$C =$

4. Suppose that D, x, y and z are integers with $D > x > y > z$. If D, x, y and z satisfy the equation $3^D - 3^x + 3^y - 3^z = \frac{1000C + 2}{9}$, determine the value of D .
假設 D, x, y 和 z 均為整數，其中 $D > x > y > z$ 。若 D, x, y 和 z 滿足等式 $3^D - 3^x + 3^y - 3^z = \frac{1000C + 2}{9}$ ，求 D 的值。

$D =$

Hong Kong Mathematics Olympiad (2018/19)
Finals (Individual – Event 3)

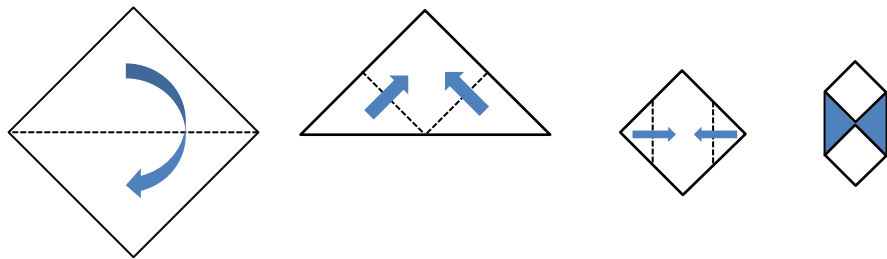
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Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. A square piece of paper of area 100 cm^2 , is folded in half along the dotted line as shown below. If the area of the shaded region in the last figure is $s \text{ cm}^2$, determine the value of s .

一張正方形紙的面積為 100 cm^2 ，按照圖中的虛線和箭咀的方向對摺。若圖中的陰影部份為 $s \text{ cm}^2$ ，求 s 的值。



$s =$

2. Suppose that $s = \frac{A}{B}$ is in simplest form. If c and d are the numbers of positive factors of A and B respectively, determine the value of $t = c + d$.

假設 $s = \frac{A}{B}$ 為最簡分數。若 c 和 d 分別為 A 和 B 的正因數的數量，求 $t = c + d$ 的值。

$t =$

3. If today is the 2nd day of a week, and known that t^{2019} days later is the u th day of that week, determine the value of u .

若今天是某一週的第 2 日，以及已知 t^{2019} 日後的當天為該週的第 u 日，求 u 的值。

$u =$

4. If v is the remainder of $1231234_{u+5} \div 123_{u+5}$, determine the value of v .
若 v 為 $1231234_{u+5} \div 123_{u+5}$ 的餘數，求 v 的值。

$v =$

Hong Kong Mathematics Olympiad (2018/19)
Finals (Individual – Event 4)

FOR OFFICIAL USE

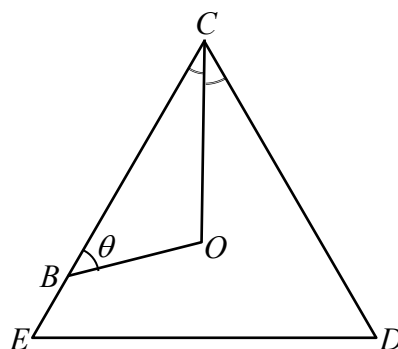
Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
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			Total score			<input type="text"/>

Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. $\triangle CDE$ is an equilateral triangle. Point O is inside $\triangle CDE$. If point B is on CE , $\theta = \angle CBO$, $\angle DCE$ is bisected by OC , and $OC : OB = 5 : 4$, determine the value of $\alpha = \sin \theta$.

$\triangle CDE$ 為一個等邊三角形。點 O 在 $\triangle CDE$ 內。若點 B 在 CE 上， $\theta = \angle CBO$ ， OC 為 $\angle DCE$ 的角平分線，以及 $OC : OB = 5 : 4$ ，求 $\alpha = \sin \theta$ 的值。



$\alpha =$

2. Suppose that there exists a function $f(x)$ defined for all integers $x \neq 0$ such that $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $f(2) = -1$. If $\beta = f\left(\frac{\alpha}{10}\right)$, determine the value of β .

假設對於任何整數 $x \neq 0$ ，函數 $f(x)$ 均滿足 $f\left(\frac{x}{y}\right) = f(x) - f(y)$ 和 $f(2) = -1$ 。

若 $\beta = f\left(\frac{\alpha}{10}\right)$ ，求 β 的值。

$\beta =$

3. If $B = \gamma p + 2\gamma(1-p)$ and $p = \frac{\beta(\gamma-40)}{100}$, determine the value of γ such that B achieves the maximum.

若 $B = \gamma p + 2\gamma(1-p)$ 以及 $p = \frac{\beta(\gamma-40)}{100}$ ，當 B 取最大值時，求 γ 的值。

$\gamma =$

4. Suppose that a , b and c are positive factors of γ with $a < b < c < \gamma$ and $ab = c$. If $x + y = a$, $x + 2y + z = b$, $y + 2z + t = c$ and $\delta = x + y + z + t$, determine the value of δ .

已知 a , b 和 c 是 γ 的正因數，而且 $a < b < c < \gamma$ 及 $ab = c$ 。若 $x + y = a$ ， $x + 2y + z = b$ ， $y + 2z + t = c$ 及 $\delta = x + y + z + t$ ，求 δ 的值。

$\delta =$